Indices

You should already know:

- how to simplify expressions such as $(a^2b)^2$
- How to calculate values such as 3^{-2} or $4^{\frac{1}{2}}$

Now we will focus on working out the value of more complicated expressions using the rules of indices

Some important index rules…

1. $x^a \times x^b \equiv x^{a+b}$
2. $x^a \div x^b \equiv \frac{x^a}{x^b} \equiv x^{a-b}$
3. $(x^a)^b \equiv x^{ab}$
4. $x^a \times y^a = (xy)^a$
5. $\frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$
6. $x^{-n} \equiv \frac{1}{x^n}$
7. $x^{\frac{1}{n}} \equiv \sqrt[n]{x}$

Examples

1. Find the value of t for which $8^t = \frac{1}{16}$.

You need to express 8 and 16 as powers of 2.

$8 = 2^3$ and $16 = 2^4$

So the equation $8^t = \frac{1}{16}$ can be written as $(2^3)^t = \frac{1}{2^4}$

Use rules **3** and **6** $2^{3t} = 2^{-4}$

Base numbers are equal, so equate indices:

$$3t = -4$$

$$t = -\frac{4}{3}$$

Examples

2. Express $\frac{6\sqrt{y}+1}{2y^2}$ in the form $3y^p + qy^r$, where p, q and r are constants to be found.

You need to split the fraction up into two terms:

$$\frac{6\sqrt{y}+1}{2y^2} = \frac{6\sqrt{y}}{2y^2} + \frac{1}{2y^2}$$

Use rule **7**

$$\frac{6y^{\frac{1}{2}}}{2y^2} + \frac{1}{2y^2}$$

Separate the number fractions from the algebraic:

$$\frac{6}{2} \times \frac{y^{\frac{1}{2}}}{y^2} + \frac{1}{2} \times \frac{1}{y^2}$$

Use rules **2** and **6**

$$3y^{\frac{1}{2}-2} + \frac{1}{2}y^{-2}$$

Simplify:

$$3y^{-\frac{3}{2}} + \frac{1}{2}y^{-2}$$

So, $p = -\frac{3}{2}$, $q = \frac{1}{2}$ and $r = -2$

Key point

$$x^{\frac{a}{b}} = (\sqrt[b]{x})^a = \sqrt[b]{x^a}$$

Calculating this form is usually easier: $(\sqrt[b]{x})^a$, because square rooting a number makes it smaller, so the calculation becomes simpler.

Practice

1. Express each of these in the form 2^n where n is an integer
   a) $2^3 \times 2^4$
   b) $(2^3)^3$
   c) 4^5
   d) $(2^4 \times 4^2)^3$

2. Express each of these numbers in the required form
   a) 4^{-2} in the form 2^n
   b) 2^{-6} in the form 4^n
   c) $8^{\frac{4}{3}}$ in the form 4^n
   d) $27^{\frac{5}{3}}$ in the form 9^n
   e) $16^{-\frac{1}{2}}$ in the form 8^n
   f) $64^{-\frac{4}{3}}$ in the form 16^n

3. By writing 16 as a power of 2, or otherwise, solve the equation $16^x = 32$

4. Solve these equations
   a) $8^x = 16$
   b) $16^x = 64$
   c) $9^x \times 3^x = 9$
   d) $\frac{8^x}{4^{x+1}} = 32$

5. Express these terms in the form ax^n where a is a real number.
   a) $\frac{4x}{2x^2}$
   b) $\frac{1}{2x^3}$
   c) $3x\sqrt{x}$
   d) $\frac{\sqrt[3]{x^2}}{4}$
   e) $\frac{2}{\sqrt{x}}$
   f) $\frac{3x}{\sqrt[3]{x}}$
   g) $\frac{3\sqrt{x^3}}{6x^2}$
   h) $\frac{10x}{\sqrt[4]{x^3}}$

6. Determine whether each of these statements is true or false. Use the rules of indices to prove those you think are true, and provide an example to prove those you think are false.
   f) $a^m \times a^n = a^{2n}$
   g) $a^n \times b^n = ab^n$
   h) $a^{mn} = a^m \times a^n$
   i) $a^n \times a^{-n} = 1$
   j) $(a^n)^n = a^{2n}$

7. Express $\frac{3x^2+2}{x^2}$ in the form $ax + bx^n$

8. Express $\frac{2x^2-3x+1}{2x^2}$ in the form $a + bx^{-1} + cx^{-2}$

9. Express these as sums of powers of x
   a) $\frac{(2x+1)(x-1)}{x}$
   b) $\frac{(3x+2)^2}{x^3}$
   c) $\frac{x^2+3x-6}{\sqrt{x}}$
   d) $\frac{(2+\sqrt{x})^2}{x^3}$

Linear Graphs and Models

You should already know:

- $\boldsymbol{y = mx + c}$ is the equation of a straight line with gradient $\boldsymbol{m}$ and y-intercept $\boldsymbol{c}$
- How to identify the gradient and y-intercept of a line using its equation and/or graph
- How to tell if two lines are parallel or perpendicular

Now we will focus on sketching the graph of a straight line with equations in the form $\boldsymbol{y = mx + c}$ or $\boldsymbol{ax + by + c = 0}$

How to interpret the gradient and y-intercept of a linear model

Examples

- Make sure to write the coordinates on the graph.
- Label your axes
- Mark on ‘0’ at the origin

Sketch the line L with the equation $3y - 4x = 12$.

You need to sets of coordinates to sketch a line. We look for the points where the line intersects with the x and y axes.

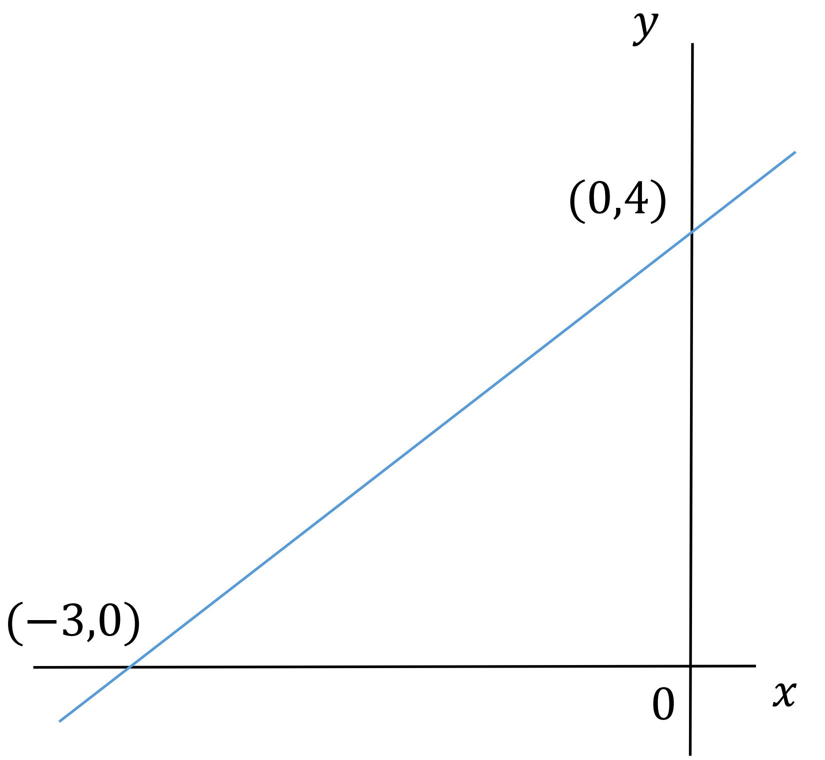
Let $x = 0$ to find the y intercept:

$$3y - 4(0) = 12$$
$$3y = 12$$
$$y = 4$$

Y-intercept: $(0,4)$

Let $y = 0$ to find the x intercept

$$3(0) - 4x = 12$$
$$-4x = 12$$
$$x = -3$$



Examples

The depth of water h cm in a bath t minutes after the plug has been removed was modelled by the straight line equation $h = 19 - 9.5t$.

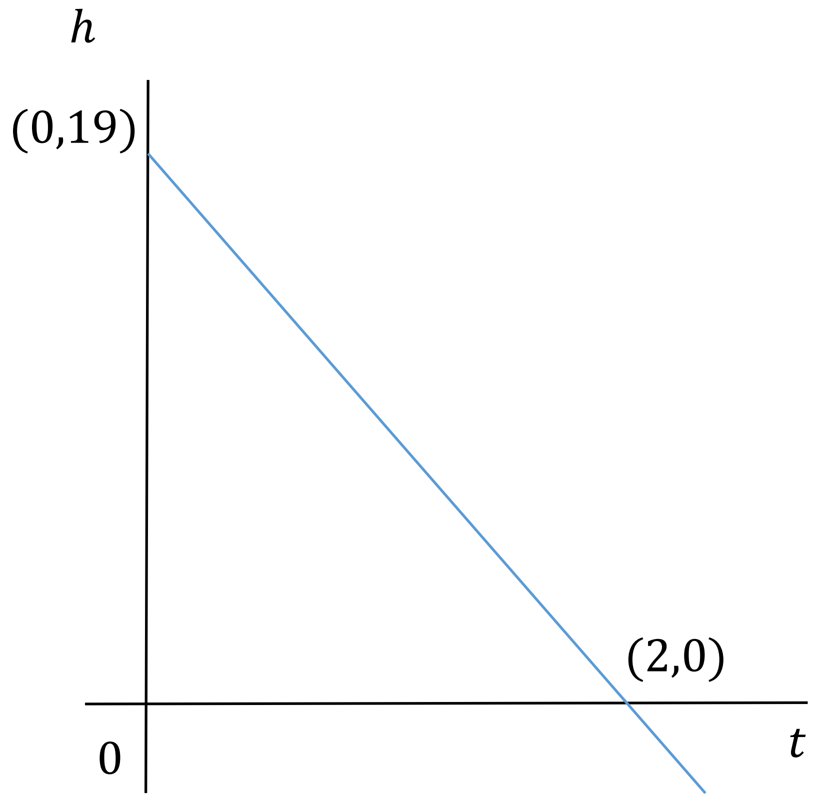
a) Sketch the graph of h against t for $t \geq 0$.

b) For this line, describe, in context

   i. What each axis crossing point represents

   ii. What the gradient represents

c) Explain why this model is not valid for $t > 2$



Compare this line equation to the standard form:

$$y = mx + c$$

$$h = -9.5t + 19$$

So we can see that h should be plotted on the y axis and t on the x axis. (See right)

Let $h = 0$

$$0 = -9.5t + 19$$

$$9.5t = 19$$

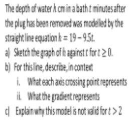
$$t = 2$$

Let $t = 0$

$$h = -9.5(0) + 19$$

$$h = 19$$

Remember, the question said that $t \geq 0$, So don't sketch the line before $t = 0$



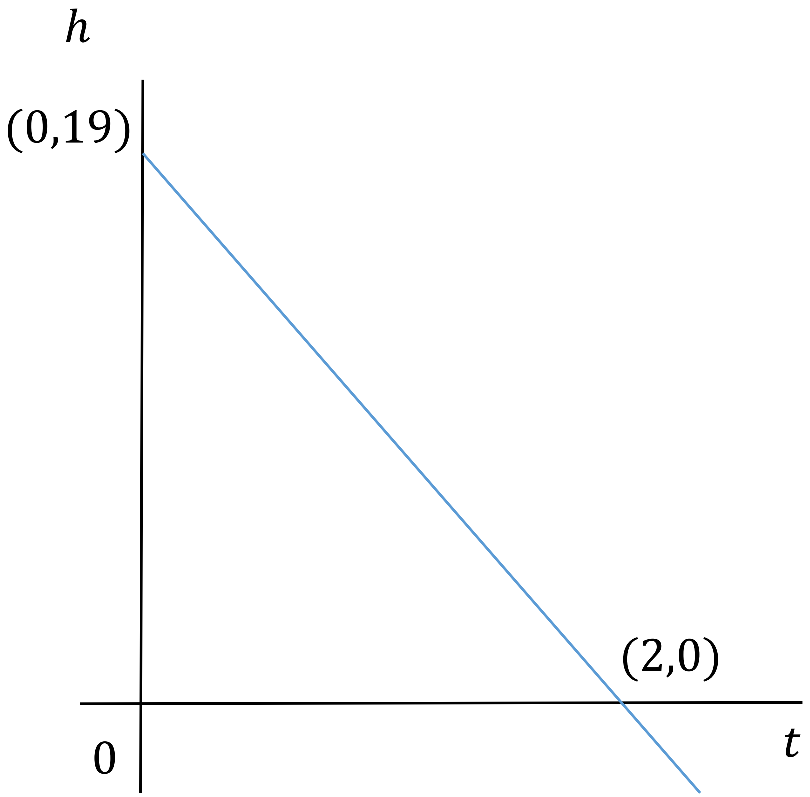
Examples

Continued…

The depth of water h cm in a bath t minutes after the plug has been removed was modelled by the straight line equation $h = 19 - 9.5t$.

b) For this line, describe, in context
   i. What each axis crossing point represents
   ii. What the gradient represents

c) Explain why this model is not valid for $t > 2$



b(i) The h intercept $(0,19)$ shows the depth of the bath tub when the plug is removed (i.e. when time = 0.)

The t intercept $(2,0)$ shows that it took 2 minutes to empty the bath (i.e. for the depth of water to fall to 0)

b(ii) The gradient is negative. This means that the depth of water was decreasing at a rate of 9.5cm per minute.

c) When $t > 2$, the depth becomes negative. This has no physical meaning. A negative depth is not valid.

Practice

1. Find the gradient m and y-intercept c of each of these lines, and sketch the lines for each of these equations. Label the points where the line crosses each axis with their coordinates.
   a) $y = 3x + 6$
   b) $y = 2 - 4x$
   c) $y = \frac{4x-5}{2}$
   d) $y = -\frac{1}{3}(3 + 4x)$
2. By making y the subject of each question, find the gradient m and y intercept c of these lines.
   a) $y - 2x + 1 = 0$
   b) $2y - 3x = 2$
   c) $4x - 3y = 1$
   d) $\frac{y}{4} + \frac{x}{2} = 3$
3. The height h (cm) of a sunflower t days after being planted is modelled by the equation $h = 3.5t + 10$.
   a) Sketch the graph of h against t for $t \geq 0$
   b) Explain, in context, what each constant in the equation $h = 3.5t + 10$ represents.
   c) When fully grown, the sunflower is expected to be 5 metres tall. According to this model, how many weeks from being planted does it take for the sunflower to reach its full height.
   d) Give one reason why this model may not be appropriate.
4. The speed v (in metres per second) of an athlete t seconds after crossing the finish line in a 100 metre race is modelled by the equation $v = 7 - 1.4t$
   a) Sketch the graph of v against t, for $t \geq 0$, labelling the axis-crossing points with their coordinates.
   b) Describe, in context,:
      i. What each axis crossing point represents
      ii. What the gradient represents
   c) According to this model, calculate the total distance an athlete runs from the start of the race until he stops.

Geometry Problems with Line Equations

You should already know:

- How to find the coordinates of the mid-point of the line joining two points
- How to find the equation of a straight line from two points

Now we will focus on finding the distance between two points and problem solving with line equations

Examples

The line L passes through the points $P(1,2)$ and $Q(-5,7)$.

a) Find an equation for L, giving your answer in the form $ay + bx = c$, where a, b and c are integers.

b) Find the area of the triangle OQR where O is the origin and R is the point where this line intersects the x-axis.

a) It's always helpful to sketch the line. This means drawing a rough idea of where the two points are and joining them.

Gradient = $\frac{\Delta y}{\Delta x} = \frac{7-2}{-5-1} = \frac{5}{-6} = -\frac{5}{6}$

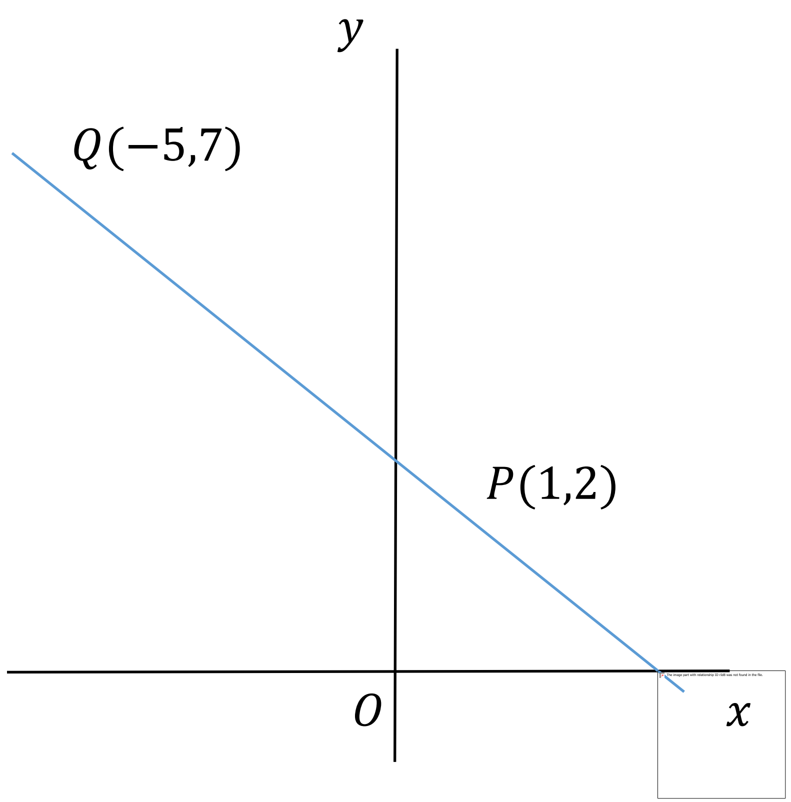
Now find the y-intercept: $y = -\frac{5}{6}x + c$

Substitute one of the sets of coordinates: $2 = -\frac{5}{6}(1) + c$

$$2 + \frac{5}{6} = c$$

$$c = \frac{17}{6}$$

$$y = -\frac{5}{6}x + \frac{17}{6} \rightarrow 6y = -5x + 17 \rightarrow 6y + 5x = 17$$



Examples

Continued...

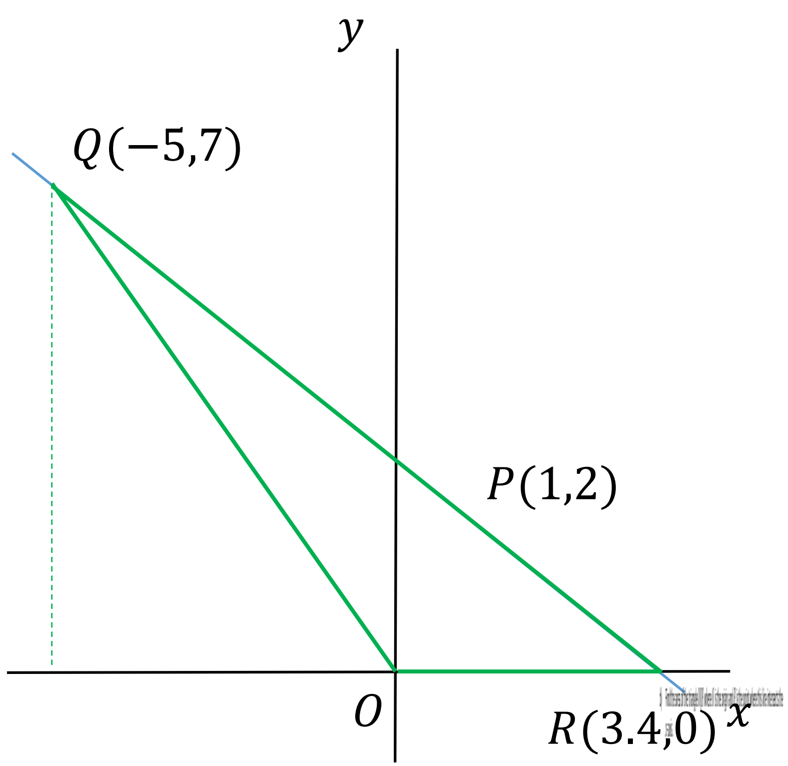
b) Find the area of the triangle OQR where O is the origin and R is the point where this line intersects the x-axis.

Let's start by finding R. It lies on the x-axis, so we know that $y = 0$.

$$6(0) + 5x = 17$$
$$5x = 17$$
$$x = 3.4$$
$$R(3.4,0)$$

Area of the triangle = $\frac{1}{2} \times base \times height$

$= \frac{1}{2} \times 3.4 \times 7 = 11.9$ units squared



- The sketch is important – it can help you correctly identify the base and height
- This is an oblique triangle, the height is still the length straight down from the top point (dotted line)

Examples

To find the distance between

$$(x_1, y_1), (x_2, y_2)$$

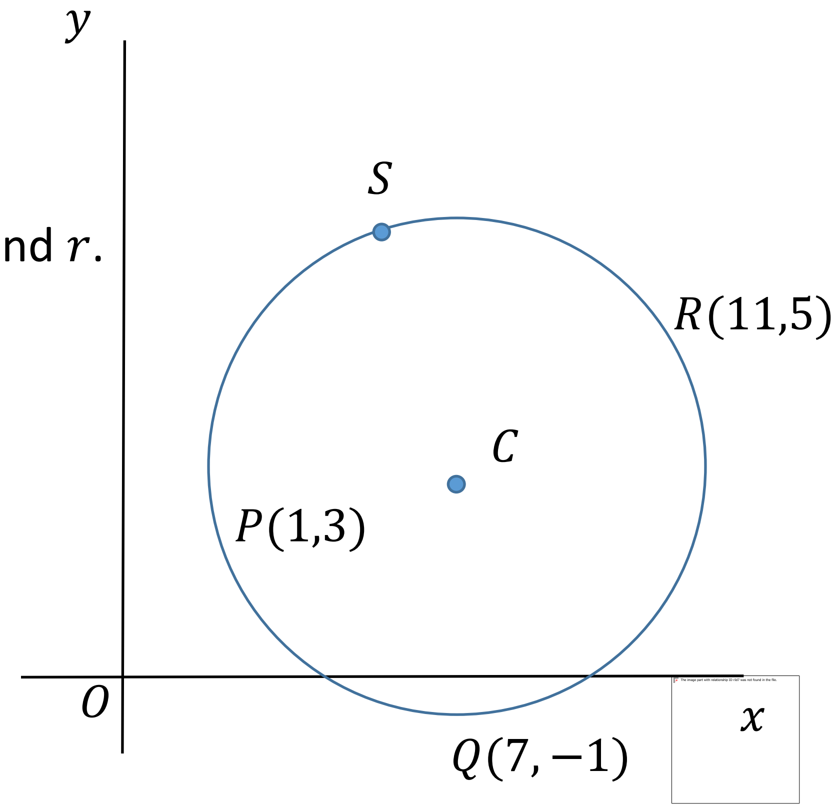
$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The diagram shows a circle, centre C which passes through the points $P(1,3)$, $Q(7,-1)$, $R(11,5)$ and S.

The lines PR and QS are diameters of this circle.

a) Find the area of this circle.

b) Find the coordinates of the point S.



a)

$$Area = \pi r^2$$

We need to find the length of PR, because it's the diameter. Then we can find r.

$$PR = \sqrt{(11-1)^2 + (5-3)^2} = \sqrt{104} \text{ (=the diameter)}$$

$$\therefore \textit{ the radius } = \frac{1}{2}\sqrt{104}$$

$$Area = \pi\left(\frac{\sqrt{104}}{2}\right)^2 = \pi\left(\frac{104}{4}\right) = 26\pi \ (\approx 81.7) \text{ units squared}$$

Examples

Continued...

The diagram shows a circle, centre C which passes through the points $P(1,3)$, $Q(7,-1)$, $R(11,5)$ and S.

The lines PR and QS are diameters of this circle.

b) Find the coordinates of the point S.

To find the midpoint, you need to find the 'average' of the coordinates
So the midpoint of two points:
$(x_1, y_1), (x_2, y_2)$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

b) The key to this is finding the coordinates of the centre C. The centre is always at the midpoint of the diameter, so let's use the points P and R:

$$Midpoint\ of\ PR = \left(\frac{1+11}{2}, \frac{3+5}{2}\right) = C(6,4)$$

Now the same should be true for QS – its midpoint should be C.

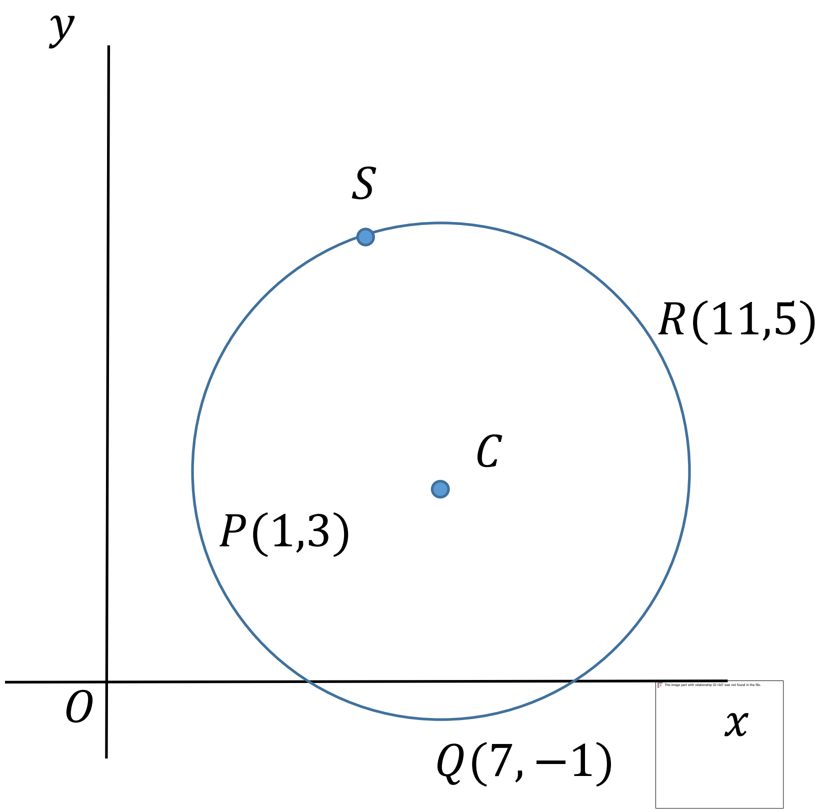
Let's give S some algebraic coordinates: $S(x, y)$

$$Midpoint\ of\ QS = \left(\frac{7+x}{2}, \frac{-1+y}{2}\right) = C(6,4)$$

$$\frac{7+x}{2} = 6 \rightarrow 7 + x = 12 \rightarrow x = 5$$

$$\frac{-1+y}{2} = 4 \rightarrow -1 + y = 8 \rightarrow y = 9$$

So, $S(5,9)$



Practice

1. The line L passes through the points $P(-4,-3)$ and $Q(4,9)$. This line crosses the y-axis at the point A and the x-axis at the point B.
   a) Find an equation for L.
   b) Find the area of the triangle OAB, where O is the origin.
2. A line passes through the points $S(3,-2)$ and $T(12,-14)$. This line crosses the y-axis at the point B. Show that the distance $AB = \frac{5}{2}$.
3. The line L has equation $2y - 4x + k = 0$ where k is a constant. The point $A\left(\frac{5}{2},\frac{1}{2}\right)$ lies on L.
   a) Show that $k = 9$.
   b) Find the y-intercept of L.
   c) Find the area of the triangle formed by this line and the two coordinate axes.
4. The line L_1 passes through the points $A(-3,-1)$ and $B\left(1,\frac{5}{3}\right)$.
   a) Show that the equation of the line L_1 can be written as $3y = 2x + 3$.
   b) The line L_2 has equation $3x + 2y = 28$. Sketch on a single diagram, the lines L_1 and L_2.
   c) Verify that point $P(6,5)$ lies on both L_1 and L_2.
   d) Find the area of the triangle formed by these lines and the y −axis.
5. Find the coordinates of the mid-point of the line AB
   a) $A(2,5), B(10,3)$
   b) $A(5,-1), B(-1,7)$
   c) $A(-6,11), B(3,4)$
   d) $A\left(\frac{3}{2},\frac{5}{3}\right), B\left(\frac{5}{2},\frac{1}{6}\right)$
6. Points $A(p,3)$ and $B(14,q)$, where p and q are constants, are such that the mid-point of AB is the point $C(8,11)$. Point D is the mid-point of AC.
   a) Show that $p = 2$ and find the value of q.
   b) Find the coordinates of D.
   c) It is given that the distance $AD = 5$. Find the distance DB.

Solving Simple Cubic Equations

You should already know:

- **How to factorise quadratic expressions**
- **How to solve a quadratic equation by factorising, the formula or completing the square**

Now we will focus on apply these skills to solving simple cubic equations

Examples

Solve the cubic equation $(\boldsymbol{x}-\mathbf{4})(\mathbf{3}\boldsymbol{x}+\mathbf{1})(\mathbf{5}-\boldsymbol{x})=\mathbf{0}$

Well, anything of the form $a \times b \times c = 0$, can be solved by understanding that a, b, or c must be 0 (or 2/all of them are 0).

So, we can say that:

$x - 4 = 0$, so $x = 4$

Or

$3x + 1 = 0 \rightarrow 3x = -1$, so $x = -\frac{1}{3}$

Or finally,

$5 - x = 0$, so $x = 5$

Examples

Solve $2x^3 - 11x^2 + 5x = 0$

This equation has a common factor of x, so it becomes easier to factorise:

$$x(2x^2 - 11x + 5) = 0$$

Now let's factorise the quadratic expression:

$$x(2x - 1)(x - 5) = 0$$

So, $x = 0$ is one option

Or, $2x - 1 = 0 \rightarrow 2x = 1 \therefore x = \frac{1}{2}$

Finally $x - 5 = 0$, $x = 5$

Practice

Solve these cubic equations

1. $(x+4)(x+2)(x-3)=0$
2. $x(x+3)(x-2)=0$
3. $(3x-2)(x+2)(5-x)=0$
4. $(x+1)(x^2-5x+6)=0$
5. $x^3+3x^2-4x=0$
6. $4x^3-3x^2=0$
7. $3x^3-10x^2-8x=0$
8. $4x^3+9x=12x^2$

Solving Simultaneous Equations

You should already know:

- **How to solve linear simultaneous equations**
- **How to solve simple simultaneous equations with one linear and one quadratic**

Now we will focus on applying these skills to finding intersections of lines and curves and solving slightly more complex equations

Examples

Find the coordinates of the points of intersection between the line $x - 2y = 1$ and the circle $x^2 + 3x + y^2 = 4$

We need to rearrange the line equation to make y or x the subject.
x would be easier.

$$x - 2y = 1$$
$x = 1 + 2y$ (Add $2y$ to each side)

Now, our aim is to form an equation with only **one** variable (all x or all y)

Substitute the rearranged line equation into the circle equation:

$$(1 + 2y)^2 + 3(1 + 2y) + y^2 = 4$$
$$(1 + 2y)(1 + 2y) + 3 + 6y + y^2 = 4$$
$$1 + 4y + 4y^2 + 3 + 6y + y^2 - 4 = 0$$
$$5y^2 + 10y = 0$$

Now solve this quadratic equation:

$$5y(y + 2) = 0$$
So $5y = 0$ or $y + 2 = 0$
$$y = 0, y = -2$$

Now find the x values. It's often better to use the line equation, because the circle may give you extra answers which aren't valid.

$$x = 1 + 2y$$
$$x = 1 + 2(0) = 1$$
Or $x = 1 + 2(-2) = 1 - 4 = -3$

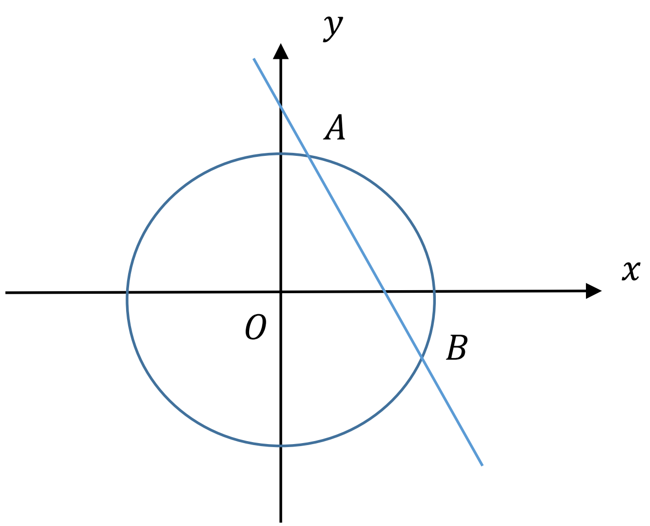
So the coordinates are:
$(1,0)$ & $(-3,-2)$

Examples

The diagram shows the circle $x^2 + y^2 = 5$ and the line $y = 3 - 2x$. Points A and B are where the line and the circle intersect.

a) Find the coordinates of the point A and the point B.

b) Show that the distance AB is $\frac{8\sqrt{5}}{5}$ units.



a) We need to solve the equations simultaneously to find where they intersect.

Substitute the line equation into the circle:

$$x^2 + (3-2x)^2 = 5$$
$$x^2 + (3-2x)(3-2x) = 5$$
$$x^2 + 9 - 12x + 4x^2 - 5 = 0$$
$$5x^2 - 12x + 4 = 0$$
$$(x-2)(5x-2) = 0$$

$$x = 2$$
$$x = \frac{2}{5}$$

Now find the y coordinates.

$$y = 3 - 2(2) = 3 - 4 = -1$$
$$y = 3 - 2\left(\frac{2}{5}\right) = 3 - \left(\frac{4}{5}\right) = \frac{11}{5}$$

$$(2, -1)\ \&\ \left(\frac{2}{5}, \frac{11}{5}\right)$$

b) Now we need to use Pythagoras' theorem:

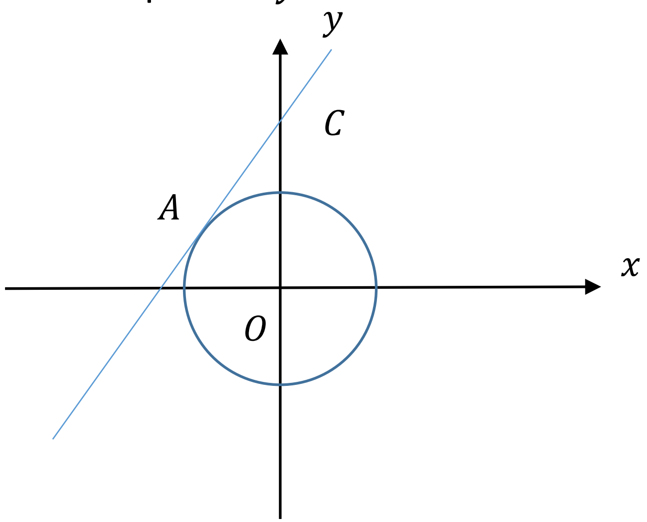
$$\sqrt{\left(\frac{2}{5} - 2\right)^2 + \left(\frac{11}{5} - (-1)\right)^2}$$

$$\sqrt{\left(-\frac{8}{5}\right)^2 + \left(\frac{16}{5}\right)^2} = \sqrt{\frac{64}{5}} = \frac{\sqrt{64}}{\sqrt{5}}$$

$$\frac{8}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$$

Practice

1. Find the coordinates of the points of intersection between these lines and curves:
   a) $y - 2x + 1 = 0$ & $y^2 - x^2 + 2x = 9$
   b) $x - 2y - 1 = 0$ & $x^2 + 4y^2 = 5$
   c) $2x + 3y + 1 = 0$ & $4x^2 - y^2 = 15$
2. The diagram shows the circle $x^2 + y^2 = 20$ and the line with the equation $y = 2x + 10$.



The line crosses the y-axis at the point C and is tangent to the circle at the point A.

a) Write down the coordinates of the point C.
b) Find the coordinates of the point A.
c) Find the area of the triangle OAC

3. A circle has the equation $x^2 + y^2 = 10$ and a line has equation $y + 3x = 10$.
   a) Find the coordinates of any points where this line and circle intersect.
   b) What information about this line and circle does your answer to **a** give?

4. A circle has equation $x^2 - 2x + y^2 = 4$ and a line has equation $2y + x = 7$.
   a) Show that the simultaneous equations $x^2 - 2x + y^2 = 4$ and $2y + x = 7$ have no real solutions.
   b) What information about this line and circle does your answer to **a** give?

Transformations of Graphs

You should already know:

- **How the transformations represented by the following affect a graph:** $-f(x), f(-x), f(x) \pm a$ & $f(x \pm a)$

Now we will extend this to stretches and learn how to keep track of the important points on a curve as it undergoes a transformation

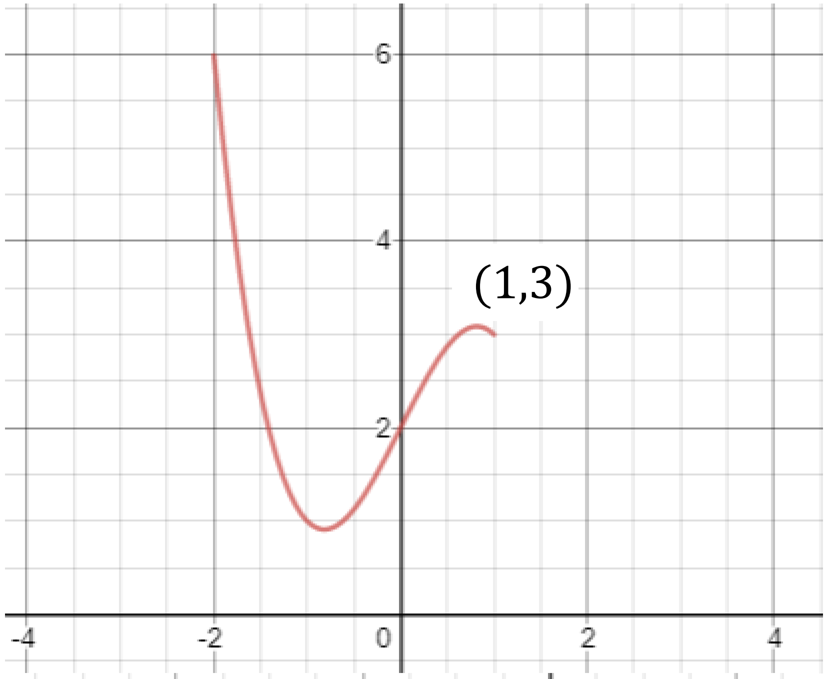
Examples:

GCSE REVIEW

The diagram shows the graph of the curve $y = f(x)$. Sketch the graph of:

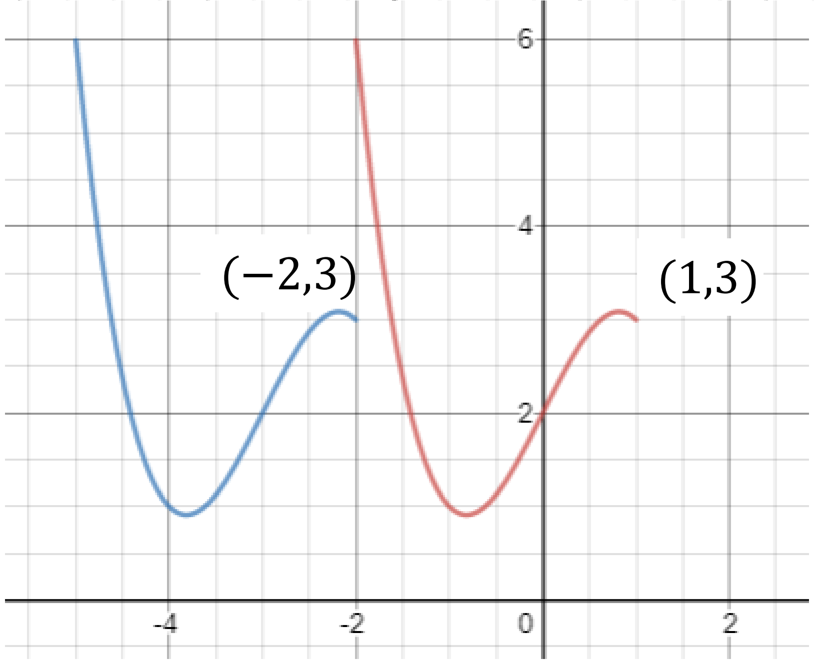
a) $y = f(x + 3)$

b) $y = -f(x)$



a) $y = f(x + 3)$. This is a translation by the vector $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$. That means it shifts to the left by 3 units.

Now we need to identify what happens to the labelled point. It moves to the left, so it should only influence the x coordinate. To move a point 3 to the left, subtract 3 from the x coordinate.



Examples:

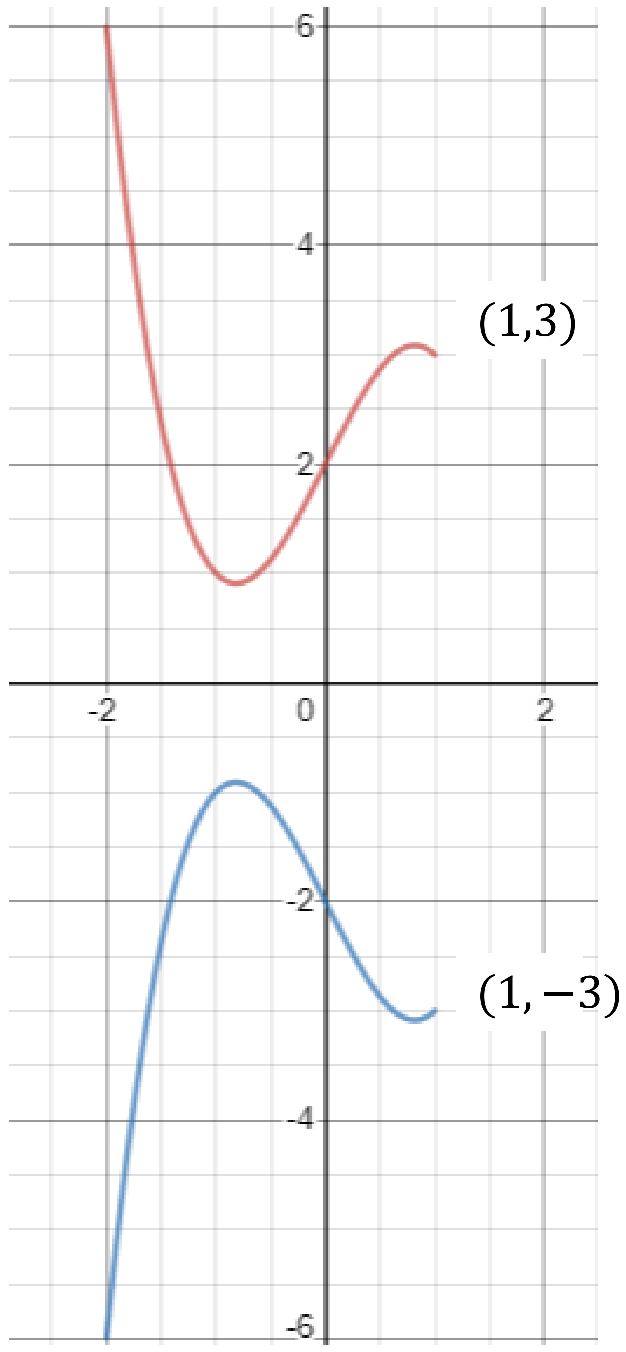
GCSE REVIEW

The diagram shows the graph of the curve $y = f(x)$. Sketch the graph of:

$b)\quad y = -f(x)$

$b)\quad y = -f(x)$. This is a reflection in the x-axis. This means that the mirror line is the horizontal axis.

Now we need to identify what happens to the labelled point. It is reflect in the x axis, so it's only its vertical position which changes (i.e. it doesn't move left or right). To reflect something the x axis, change the sign of its y coordinates.



Stretches and Compressions

- The graph of $y = af(x)$ is a stretch of $y = f(x)$ parallel to the y-axis, scale factor a

What does this mean?
It means if you see $y = 3f(x)$, then the graph will be stretched vertically by a scale factor of 3. (i.e. multiply all the y-coordinates by 3)

- The graph of $y = f(ax)$ is a stretch of $y = f(x)$ parallel to the x-axis, scale factor $\frac{1}{a}$

What does this mean?
It means if you see $y = f(3x)$, then the graph will be compressed or squashed horizontally by a scale factor of 3. (i.e. divide all the x-coordinates by 3)

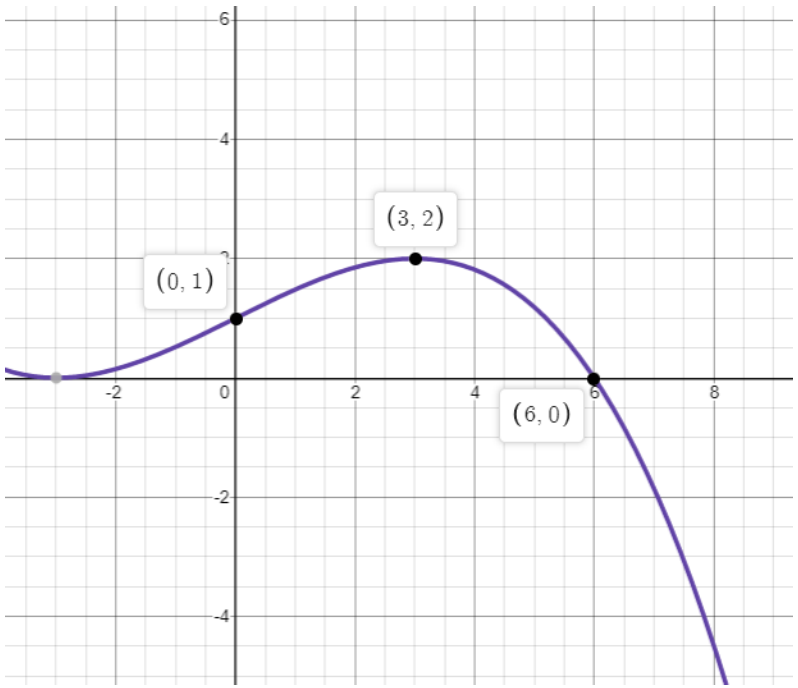
Examples:

The diagram shows the graph of $y = f(x)$. The graph has a maximum point at $(3,2)$ and crosses the axes at $(0,1)$ and $(6,0)$. Sketch, on separate diagrams, the curve with equation:

$a)\ \ y = 2f(x)$

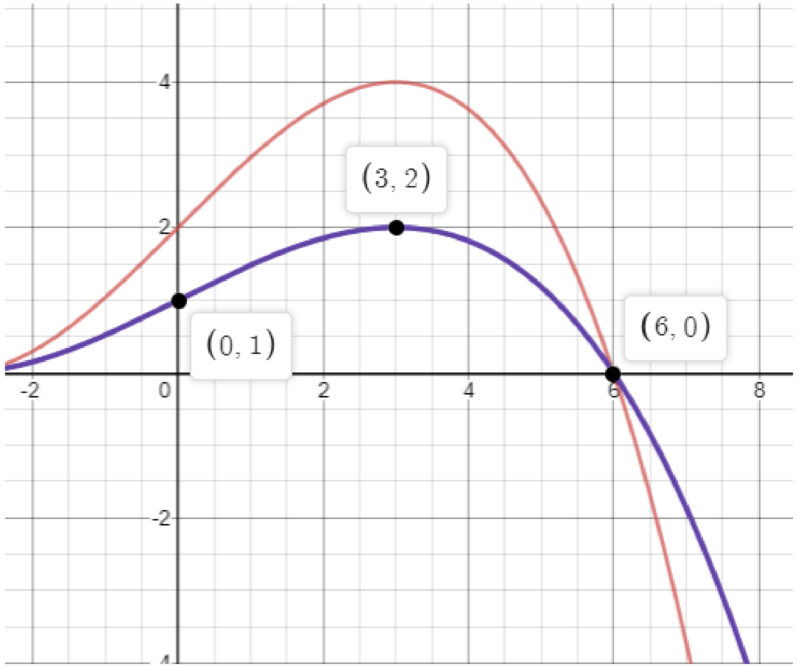
$b)\ \ y = f(3x)$

Showing clearly the new coordinates of the three labelled points.



$a)\ \ y = 2f(x)$

This means a stretch vertically (parallel to the y-axis) by a scale factor of 2. So, we need to multiply all the y coordinates by 2, but leave the x coordinates unchanged.



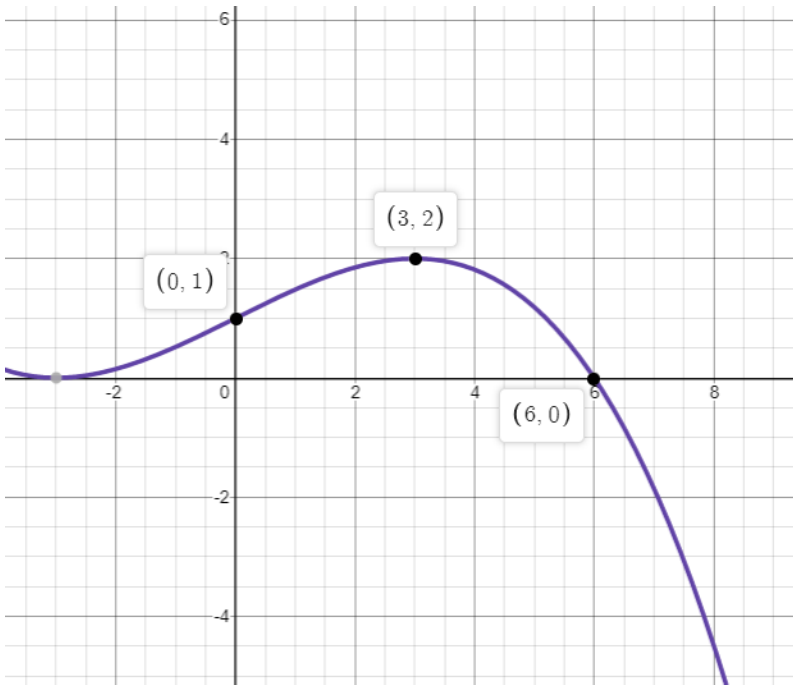
Notice that the graph stretches up and down. Imagine pulling it in both directions, not just up. This is why it is above the original curve on top, but below the original curve underneath the axis.

Examples:

The diagram shows the graph of $y = f(x)$. The graph has a maximum point at $(3,2)$ and crosses the axes at $(0,1)$ and $(6,0)$. Sketch, on separate diagrams, the curve with equation:

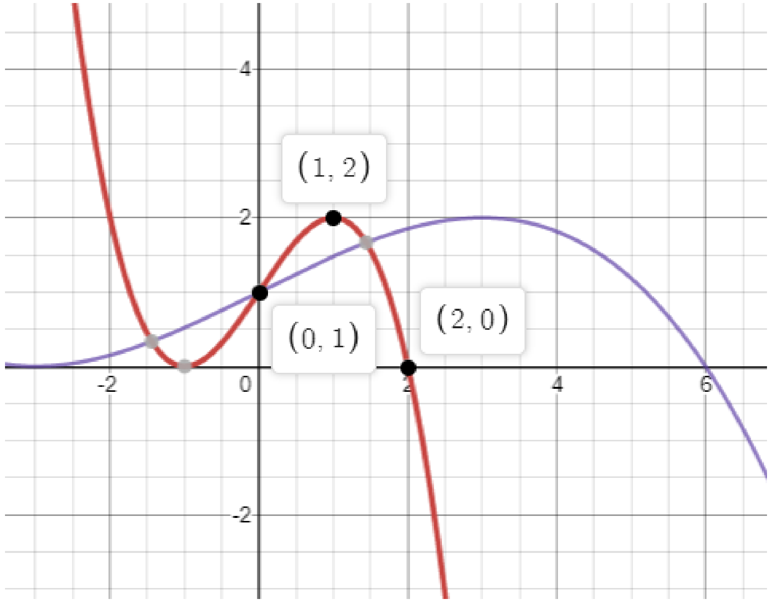
$b)\ y = f(3x)$

Showing clearly the new coordinates of the three labelled points.



$b)\ y = f(3x)$

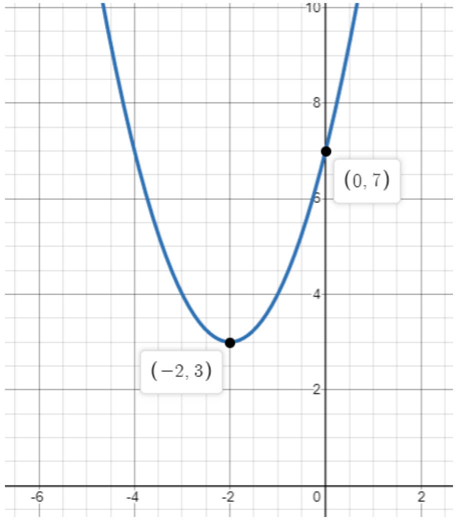
This means a stretch horizontally (parallel to the x-axis) by a scale factor of $\frac{1}{3}$. So, we need to divide all the x coordinates by 3, but leave the y coordinates unchanged.



Notice that the graph seems squashed horizontally, but the heights of all the points haven't changed. Imagine pushing the graph in from the left and the right to squash it.

Practice

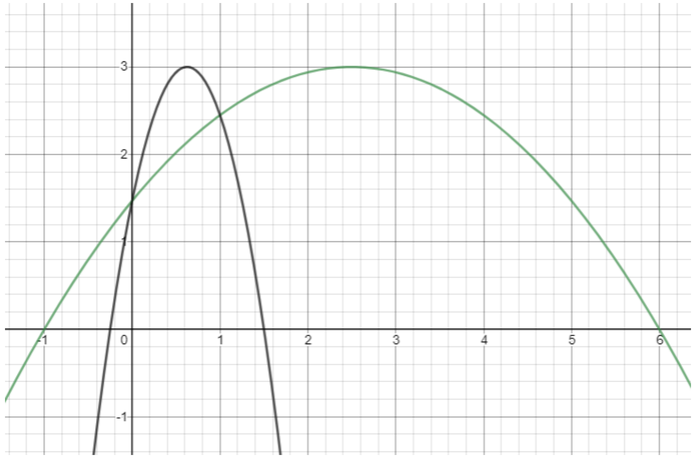
1. The diagram shows the graph $y = f(x)$ which crosses the y-axis at the point $(0,7)$. Point $P(-2,3)$ is the minimum point on this graph.



Sketch on separate diagrams the graphs below, showing the coordinates of the y intercept and minimum point.

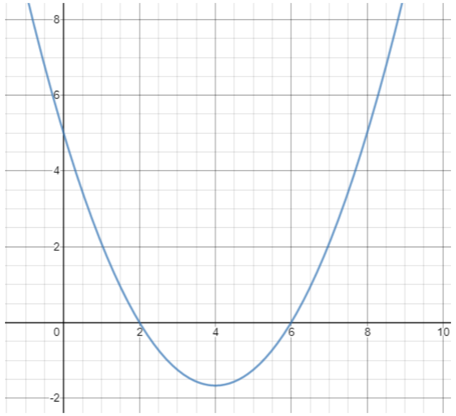
a) $f(x) + 2$
b) $y = 2f(x)$
c) $y = f(2x)$
d) $y = f(x - 2)$

2. The diagram shows the graph of $y = f(x)$ and the graph of $y = f(ax)$, where a is a positive constant. The graph $y = f(x)$ crosses the x-axis at the points where $x = -1$ and $x = 6$. Both graphs cross the y-axis at the point $(0,2)$.



a) Describe, in terms of a, the transformation which maps the graph of $y = f(x)$ onto the graph of $y = f(ax)$.
Under this transformation, the point $P(2,3)$ is mapped to the point $P'\left(\frac{1}{2}, 3\right)$.
b) Find the value of a.
c) For this value of a, find the coordinates of the points where the graph of $y = f(ax)$ cross the x-axis.
d) Sketch, on separate diagrams, the graphs of
i. $y = f(x + 2)$
ii. $y = 4f(4x)$
On each sketch, mark the axes crossing points and the maximum point with their coordinates.

3. The diagram shows the graph of $y = f(x)$. The graph crosses the y-axis at the point $(0,5)$ and the x-axis at the points $(2,0)$ and $(6,0)$. Point $P(4, -2)$ is the minimum point on this graph.



a) Sketch, on separate diagrams, the graphs of:
i. $y = f(x) + 3$
ii. $y = f\left(\frac{1}{2}x\right)$
iii. $y = -f(2x)$
iv. $y = 2f(3x)$

b) Under the translation $\begin{pmatrix} -2 \\ k \end{pmatrix}$, where $k > 0$ the graph above is mapped onto a new graph which crosses the x-axis at a single point. Find the value of k